Minimum Weight Design of Axisymmetric Sandwich Plates

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The problem of minimum weight design of laminated circular sandwich plates with axial symmetry is formulated using optimal control theory. A steepest descent algorithm is used to solve the resulting two point boundary value problem. Inequality constraints on minimum face and core thicknesses, maximum stress levels and maximum displacement are included. Both statically determinate and statically indeterminate plates are considered. Several examples are presented illustrating the general configuration of minimum weight designs and the dramatic weight savings attainable.

I. Introduction

As the demand for improved light weight structural design intensifies, structural engineers and designers are being forced to carefully consider many nontraditional design techniques and materials. The high strength and low weight characteristics of sandwich materials mark such materials as candidates for increasing numbers of applications. In the context of this paper, sandwich plates are defined as structural members comprising a core, of low density and relatively low tensile strength, bonded between two relatively thin faces made of high density, high strength material. The geometry and loading are assumed to be axially symmetric. The plates are isotropic with respect to the midplane of the plate.

Up to the present time, most applications of sandwich elements have been confined to elements with constant face and core thickness. It is clear that future design techniques will need to consider the improvements that can be attained through the use of variable thickness designs. Huang[1] and Huang and Alspaugh[2] discuss minimum weight design of sandwich beams. In this paper, we present a technique weight design of circular plates under axially symmetric loading. The thickness of the two faces are assumed equal but unknown functions of radius. The core thickness is also unknown. In optimization theory terminology, these face and core thicknesses are termed control (or design) variables. Both statically determinate and statically indeterminate axisymmetric designs are considered. The designs are restrained to lie in the elastic range of the sandwich materials. Optimum control theory is used to reduce the problem to the solution of a two point boundary value problem (TPBVP). The TPBVP is solved using a steepest descent technique. Several inequality constraints often encountered in practical problems are included.

II. Governing Equations and Boundary Conditions

In Sec. II, the derivation of the governing equations and boundary conditions is sketched. For a more complete derivation see Ref. 1. The sandwich plate is assumed to have an outside radius R and to have two face layers of equal thickness f separated by a core of thickness c. The faces are made of high strength material of high density (ρ_1); the core is made of lower strength, low density (ρ_2) material. Both materials are assumed to be isotropic. The faces are thin compared to the core, i.e., $f/c << 1$. Reissner's assumptions[3] are employed: i.e., the core carries no radial or transverse normal stresses; the faces are loaded as membranes; the local flexural rigidity of the faces is negligible. Thus, the flexural rigidity of the plate is $D = E_f f(f+c)^2/2(1-\upsilon^2)$ and the shear rigidity is $G = G_c(f+c)^2/c$. The Timoshenko sign convention for circular plates[4] is used. The transverse displacement w is assumed to be small i.e., $w/R << 1$.

As a consequence of the assumptions previously made, it is possible to partition the total deflection w into partial deflections.

$$w = w_B + w_S \tag{1}$$

The use of the partial deflections is discussed by Erickson.[5] It should be noted that w_B and w_S have no real physical significance. They are simply convenient quantities with which to deal.

We shall use variational principles to derive the governing equations and boundary conditions. The strain energy of the circular sandwich plate is

$$U = 2\pi \int_0^R \frac{D}{2} \left[\left(\frac{d^2 w_B}{dr^2} + \frac{1}{r} \frac{dw_B}{dr} \right)^2 - \frac{2(1-\upsilon)}{r} \frac{dw_B}{dr} \frac{d^2 w_B}{dr^2} \right] rdr + 2\pi \int_0^R \frac{S}{2} \left(\frac{dw_s}{dr} \right)^2 rdr \tag{2}$$

If the reaction forces do no work, the external work is

$$U_L = -\pi \int_0^R 2qwrdr \tag{3}$$

where q is the external load intensity. The total potential energy is then

$$U_p = U + U_L \tag{4}$$

The principle of minimum potential energy requires that the first variations of Eq. (4) should be zero. This yields the two Euler-Lagrange equations

$$\frac{d^2}{dr^2}\left[Dr \frac{d^2 w_B}{dr^2} + D\upsilon \frac{dw_b}{dr} \right] - \frac{d}{dr}\left[\frac{D}{r}\frac{dw_B}{dr} + D\upsilon \frac{d^2 w_B}{dr^2} \right] - qr = 0 \tag{5}$$

and

$$\frac{d}{dr}\left(Sr \frac{dw_s}{dr} \right) + qr = 0 \tag{6}$$

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We note in passing that in the special case of f and c constant, Eqs. (5 and 6) reduce to the familiar form

$$\nabla^4 w_B = \frac{q}{D} \tag{7}$$

$$\nabla^2 w_S = -\frac{q}{S} \tag{8}$$

In the general case, Eq. (5) can be integrated once to give

$$D\frac{d}{dr}\left(\frac{d^2 w_B}{dr^2}+\frac{1}{r}\frac{dw_B}{dr}\right)+\frac{dD}{dr}\left(\frac{d^2 w_B}{dr^2}+\upsilon\frac{dw_B}{dr}\right)=Q_r \tag{9}$$

where

$$Q_r = \frac{1}{r}\int_0^r qrdr \tag{10}$$

Introducing the relations

$$M_r = -D\left(\frac{d^2 w_B}{dr^2}+\frac{\upsilon}{r}\frac{dw_B}{dr}\right) \tag{11}$$

$$M_t = -D\left(\frac{1}{r}\frac{dw_B}{dr}+\upsilon\frac{d^2 w_B}{dr^2}\right) \tag{12}$$

we write Eq. (9) as

$$\frac{d}{dr}[rM_r] - M_t + rQ_r = 0 \tag{13}$$

Equation (6) can also be integrated to give

$$\frac{dw_S}{dr} = -\frac{Q_r}{S} \tag{14}$$

Setting the first variation of the potential energy to zero also yields the natural boundary conditions

a) Free edge

$$M_r = 0 \tag{15a}$$

$$Q_r = 0 \tag{15b}$$

b) Simply supported edge

$$w = w_B + w_S = 0 \text{ or } w_B = 0 \text{ and } w_S = 0$$

$$M_r = 0 \tag{16}$$

c) Clamped edge‡

$$w = w_B + w_s = 0 \text{ or } w_B = 0 \text{ and } w_S = 0$$

$$\frac{dw}{dr} - \gamma_r = 0 \text{ or } \frac{dw_B}{dr} = 0 \tag{17}$$

That the designs resulting from the optimization be physically realizable, and to insure that maximum stress levels remain within acceptable bounds, the following inequality constraints will be imposed:

a) Face thickness equal to or greater than a specified minimum value a.

$$a - f \leq 0 \tag{18}$$

b) Core thickness equal to or greater than a specified minimum value b.

$$b - c \leq 0 \tag{19}$$

c) Bending stress at any point in the plate must not exceed a given maximum stress σ_m. Since the maximum bending stress at a given radius will occur at the outside surfaces of the faces

$$\left[\frac{M_r}{f(f+c)}\right]^2 - \sigma_m^2 \leq 0 \tag{20}$$

d) Shearing stress at any point in the beam must not exceed a given maximum stress τ_m. Since the shear stress at a given radius will be maximum at the neutral surface

$$\left[\frac{Q_r}{f+c}\right] - \tau_m^2 \leq 0 \tag{21}$$

Finally, we impose the requirment that in the case of solid plates or annular plates with one free edge, the deflection at the center (or in the case of an annular plate, the free edge) shall be equal to a specified value X. This equality constraint is used in lieu of the more general, and difficult, inequality constraint

$$|w(r)| \leq X$$

The imposition of some constraint on deflection is generally necessary in physical problems since minimum weight designs without deflection constraints will be characterized by large deflections. To simplify the algorithm, the inequality constraint listed above has been replaced by the equality constraint previously mentioned. In most statically determinate cases, the location of the maximum deflection is apparent. The more general inequality constraint can be treated but will require modification of the algorithm.

Inspection of the governing equations shows that if the plate is statically determinate, the core and face thickness (c and f) appear explicitly and no derivaties of these terms appear. Thus, in these cases, c and f are control variables.

In the case of statically indeterminate plates, a somewhat different formulation will be required. This formulation is discussed in Section IV. In Secs. III to V, we discuss the formulation and solution of minimum weight design problems using optimum control theory.

III. Statically Determinate Problems

In Sec. III, optimal control theory is used to formulate the problem of mininum weight design of statically determinate sandwich plates. Readers are referred to the back-up paper of Ref. 2 for a brief discussion of optimum control theory and structural design. Further discussion of the state variable form used herein can be found in Citron[6] or other modern texts in optimal control theory.

For minimum weight design of a circular plate as shown in Fig. 1, the index of performance (or objective function, or cost function) is given by

$$IP = \int_{r_0}^{R} 2\pi r(2\rho_1 f + \rho_2 c)dr \tag{22}$$

Since it will be convenient to work with the problem stated as a "problem of Mayer" we introduce the auxilliary state variable

$$\dot{X}_5 = 2\pi r(2\rho_1 f + \rho_2 c) \tag{23}$$

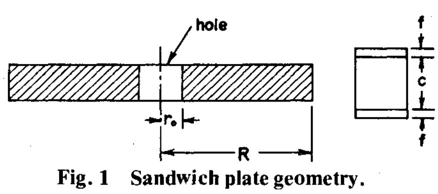

Fig. 1 Sandwich plate geometry.

‡See Ref. 5, p. 14.

where the dot implies differentiation with respect to the independent variable r. Choosing the state variables as

$$X_1 = w_b \tag{24a}$$

$$X_2 = w_S \tag{24b}$$

$$X_3 = \dot{w}_B \tag{24c}$$

$$X_4 = M_R \tag{24d}$$

and the control variables as

$$U_1 = f \qquad U_2 = c \tag{25}$$

the optimization problem is stated as follows. Choose $U_1(r)$ and $U_2(r)$ such that

$$IP = g(R) \equiv X_5(R) \tag{26}$$

is minimized subject to the differential constraints

$$\dot{X}_1 = X_3 \tag{27a}$$

$$\dot{X}_2 = -\frac{Q_r U_2}{G_c(U_1+U_2)^2} \tag{27b}$$

$$\dot{X}_3 = -\left[\frac{2(1-v^2)X_4}{E_f U_1(U_1+U_2)^2} + \frac{v}{r}X_3\right] \tag{27c}$$

$$\dot{X}_4 = -\left[Q_r + \frac{(1-v)}{r}X_4 + \frac{E_f U_1(U_1+U_2)^2}{2}\frac{X_3}{r^2}\right] \tag{27d}$$

$$\dot{X}_5 = 2\pi r(2\rho_1 U_1 + \rho_2 U_2) \tag{27e}$$

Since in this problem, f and c are control variables, the constraints of Eqs. (17-20) can be expressed as control variable inequality constraints.

$$C_1 \equiv a - U_1 \leq 0 \tag{28a}$$

$$C_2 \equiv b - U_2 \leq 0 \tag{28b}$$

$$C_3 \equiv \frac{X_4^2}{U_1^2(U_1+U_2)^2} - \sigma_m^2 \leq 0 \tag{28c}$$

$$C_4 \equiv \frac{Q_r^2}{(U_1+U_2)^2} - \tau_m^2 \leq 0 \tag{28d}$$

The optimization problem can be compactly stated in vector notation as: choose **U** such that the index of performance

$$IP = X_5(R) \tag{29}$$

is minimized subject to the state equations

$$\dot{X} = f \tag{30}$$

and the control

$$\mu^T C = 0 \tag{31}$$

In Eq. (31) the unknown multipliers μ are chosen such that if $C_i \neq 0$, $\mu_i = 0$. From optimal control theory we find that the following equations must be satisfied in the domain $r_o \leq r \leq R$,

$$\dot{X} = f \tag{32}$$

$$\dot{\lambda}^T = -\lambda^T \frac{\partial f}{\partial x} - \mu^T \frac{\partial C}{\partial x} \tag{33}$$

$$\lambda^T \frac{\partial f}{\partial U} - \mu \frac{\partial C}{\partial U} = 0 \tag{34}$$

writing Eq. (33) in explicit scalar form

$$\frac{d\lambda_1}{dr} = 0$$

$$\frac{d\lambda_2}{dr} = 0$$

$$\frac{d\lambda_3}{dr} = -\lambda_1 + \frac{v}{r}\lambda_3 + \frac{E_f U_1(U_1+U_2)^2}{2r^2}\lambda_4$$

$$\frac{d\lambda_4}{dr} = \frac{2(1-v^2)}{E_f U_1(U_1+U_2)^2}\lambda_3 + \frac{(1-v)}{r}\lambda_4 - \frac{2X_4}{U_1^2(U_1+U_2)^2}\mu_3$$

$$\frac{d\lambda_5}{dr} = 0 \tag{35}$$

Equations (34) can also easily be written in explicit form. However, to save space, they are not repeated here.

Complete specification of the TPBVB requires a set of initial and terminal conditions to be imposed upon the solution. Part of these conditions is obtained as input from the specified boundary conditions. The remaining conditions are obtained from the transversality conditions

$$dg + \lambda^T dx \Big|_{r_o}^{R} = 0 \tag{36}$$

Where the dx's are chosen to satisfy any specified boundary conditions, e.g., if $x_1(r_o) = 0$ then $dx_1(r_o) = 0$, if $X_1(R) + X_2(R) = X$ then $dX_1(R) + dX_2(R) = 0$. When all boundary conditions have been utilized, by setting the coefficient of the various differentials in Eq. (35) to zero, the correct number of auxilliary conditions is obtained. Further, it is pointed out that the variables X and λ are continuous.

We have not discussed the manner in which the optimization problem is reduced to the solution of a nonlinear two point boundary value problem. There are several techniques which can be applied to the numerical solution of TPBVP's. The results presented in this paper were obtained by solving the TPBVP using a steepest descent method. This technique is clearly described in Citron[6] and Denham.[7]

Example 1 A Statically Determinate Plate

As a first example, we consider the problem of design of an annular plate simply supported at the outside edge, free at the inner edge, and loaded with a linearly varying load. As shown in Fig. 2 in this case

$$q(r) = q_o\left(\frac{R-r}{R-r_o}\right)$$

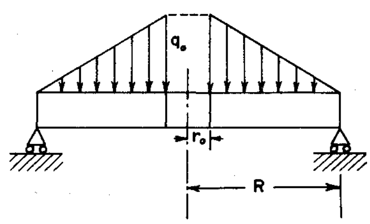

Fig. 2 Loading and support —example 1.

Since this problem is statically determinate,

$$Q_r = \frac{q_o}{(R-r_o)r}\left[\frac{R(r^2-r_o^2)}{2} - \frac{(r^3-r_o^3)}{3}\right] \tag{37}$$

The boundary conditions are

$$\begin{array}{ll} \text{at} \quad r=r_o & \text{at} \quad r=R \\ x_1+x_2=X & x_1=0 \\ x_4=0 & x_2=0 \\ x_5=0 & x_4=0 \end{array} \tag{38}$$

The following parameters were used

$$\begin{array}{ll} E_F=30\times 10^6 \text{ psi} & G_c=37\times 10^3 \text{ psi} \\ \nu=0.3 & \\ \rho_1=0.283 \text{ lb/in.}^3 & \rho_2=0.0025 \text{ lb/in.}^3 \\ r_o=1.0 \text{ in.} & R=10 \text{ in.} \\ a=0.001 \text{ in.} & b=0.1 \text{ in.} \\ \sigma_m=180{,}000 \text{ psi} & \tau_m=200 \text{ psi} \\ X=0.4 \text{ in.} & \end{array} \tag{39}$$

Three values of load intensity were used, $q_o=2$, 20, and 100 psi.

Two types of optimum design were obtained for this, and the other examples presented later. First, a constraint was introduced that caused the core thickness to be constant although unknown. Second, the constraint on constant core thickness was released and the problem of variable face and core thickness solved. Of course, the solution obtained for constant core thickness will not be superior to the design obtained when both face and core are allowed to vary.

The results of these computations are shown in Figs. 3-8. These curves are quite self-explanatory. The effect of increasing load intensity on driving the solutions onto constraint boundaries is quite evident. To compare design efficiencies, Table 1 has been prepared which indicates the minimum weight design as well as the percent improvement obtained by selecting a variable core-variable face design over a constant core-variable face design. While Figs. 3 and 4 show a steep slope in the vicinity of r_o, it should be noted that the solutions are finite near $r=r_o$.

Example 2 Statically Determinate Plate Under Uniform Load

As a second example, the minimum weight design of a uniformly distributed load was determined. The design data of Example 1 was used. For this example, the load intensity was 20 psi. The results of this design are shown in Figs. 9-12.

The minimum weight constant core-constant face thickness design for this case weighs 2.09 lb. The minimum weight constant core-variable face thickness design weighs 1.02 lb. and the variable core-variable face thickness design weighs 0.97 lb. In this case, allowing both core and faces to vary can achieve a weight reduction of over 50%. The variable core-variable face design represents a savings of 5% over the constant core-variable face design.

IV. Statically Indeterminate Plates

Analysis of the governing equations for minimum weight design of statically indeterminate sandwich plates shows that

Table 1 Weights and weight savings for example 1

Load (psi)	c = Constant (lb)	c = Variable (lb)	Weight savings (%)
2	0.404	0.349	13.50
20	0.799	0.692	13.50
100	1.381	1.320	4.50

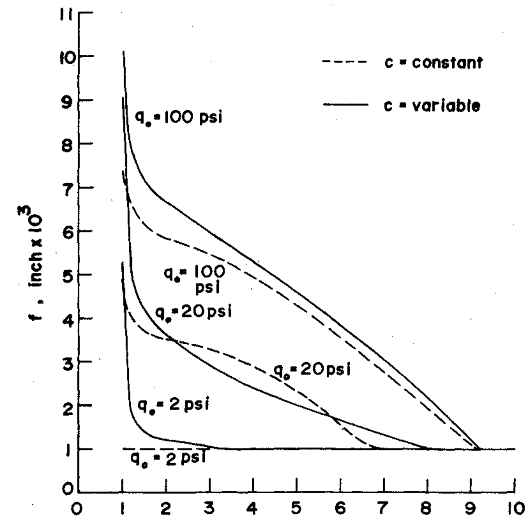

Fig. 3 Face thickness—example 1.

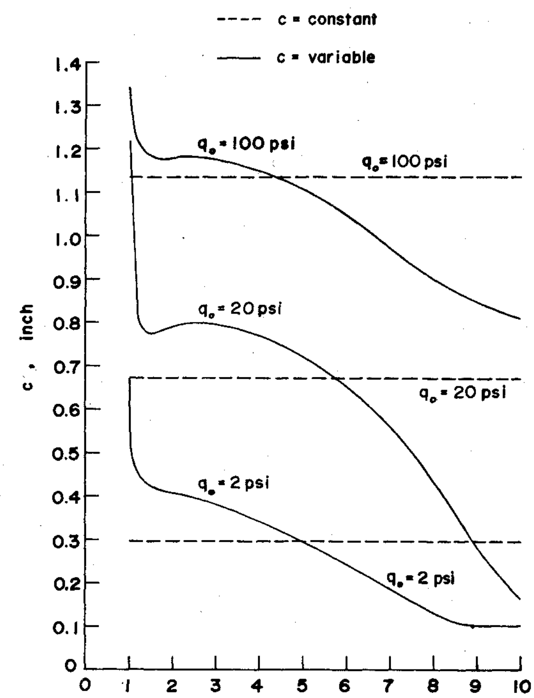

Fig. 4 Core thickness—example 1.

the technique presented in connection with statically determinate plates can be used with some minor modifications. In this paper, we treat the indeterminacy by the introduction of reaction forces of unknown magnitude. By artificially treating these reactions as state variables, i.e., introducing a state equation showing that the rate of change of the reaction with respect to radius is zero, the design problem reduces to the form previously treated.

This process is illustrated by considering the case of a reaction force P. Since the reaction will vary with the flexural and shear rigidities of the plate as well as the load magnitude, we can write

$$IP=P(x, U)$$

Introducing a state variable

$$X_6(r)=P(x, U)$$

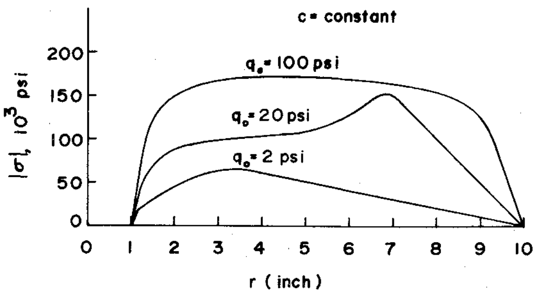

Fig. 5 Bending stress—example 1 (constant core thickness).

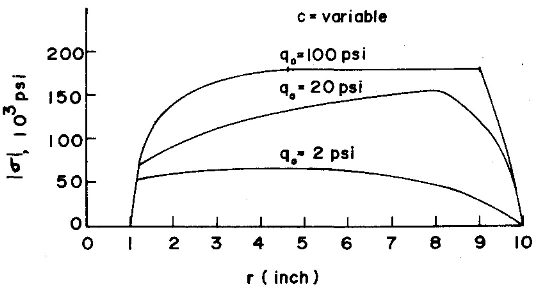

Fig. 6 Bending stress—example 1 (variable core thickness).

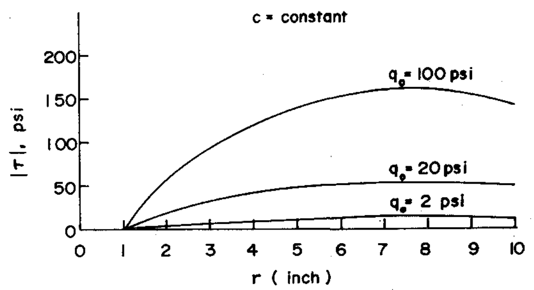

Fig. 7 Shearing stress—example 1 (constant core thickness).

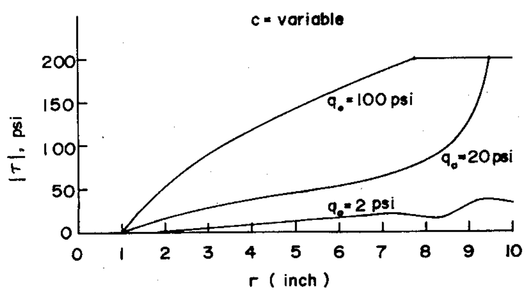

Fig. 8 Shearing stress—example 1 (variable core thickness).

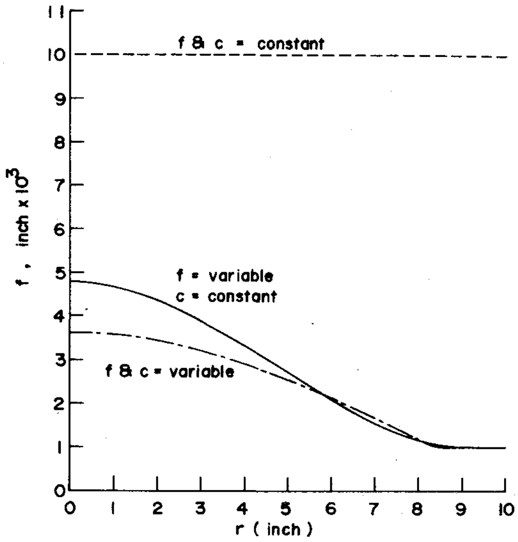

Fig. 9 Face thickness—example 2.

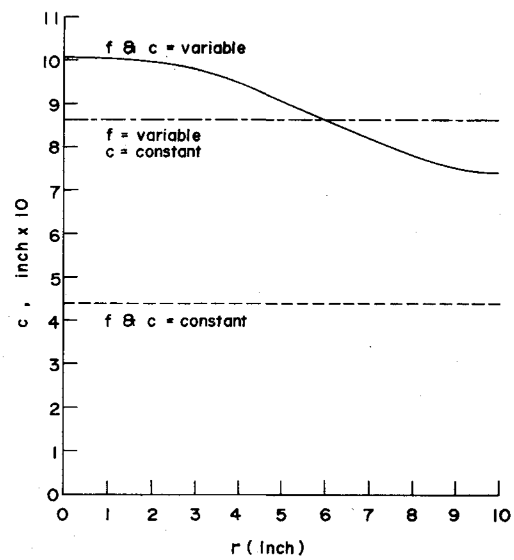

Fig. 10 Core thickness—example 2.

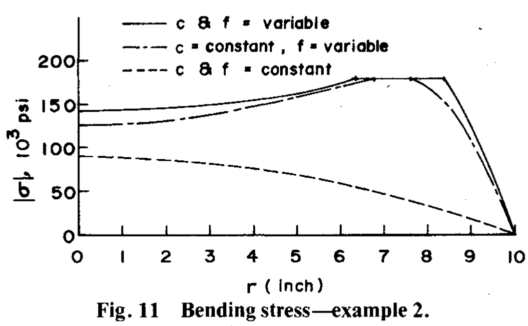

Fig. 11 Bending stress—example 2.

Since *IP* is not an explicit function of *r*, the state equation corresponding to X_6 is

$$\dot{X}_6 = 0$$

The introduction of the state variable increases the order of the system to be solved by two (an additional Lagrange multiplier is also required). The additional boundary conditions required are obtained from transversality as before. This same process is repeated for each additional unknown reaction. The moment and shear equations are modified to include the unknown reactions. The resulting TPBVP is then solved by an appropriate technique. In this paper, as noted earlier, a steepest descent method was used.

Example 3 Statically Indeterminate Plate

As an example of the applicability of the optimization methods previously described, the plate shown in Fig. 13 was designed for minimum weight. The basic dimensions and material properties used in Example 1 were also used in this example. Note that nonlinearity is introduced because of the change of character of the load terms when contact is made between the plate and center support.

The nonlinear reaction force was treated through the introduction of an artifical state variable as previously described. If contact is made, the central displacement will be given by $x_1 + x_2 = X$. Note that unlike examples in which the boundary condition is such that $X = 0$, the boundary conditions are imposed on the sum and not on the individual partial deflections.

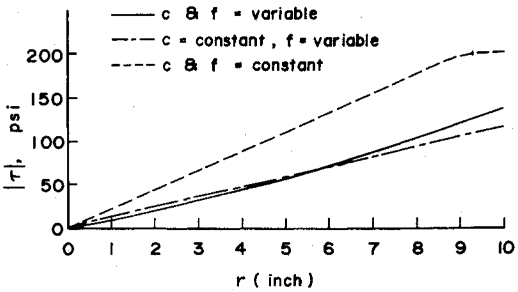

Fig. 12 Shearing stress—example 2.

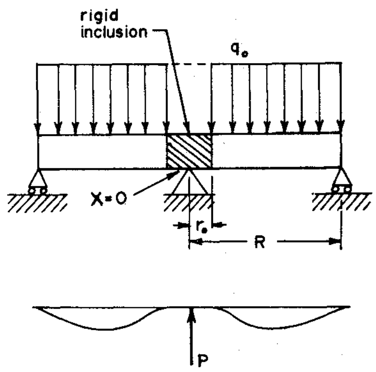

Fig. 13 Load geometry and identification of reaction force —example 3.

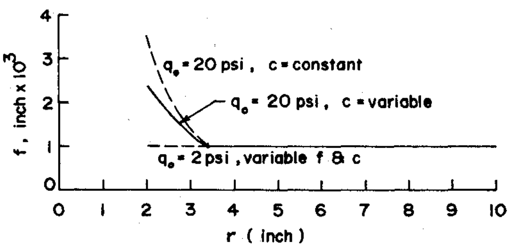

Fig. 14 Face thickness—example 3.

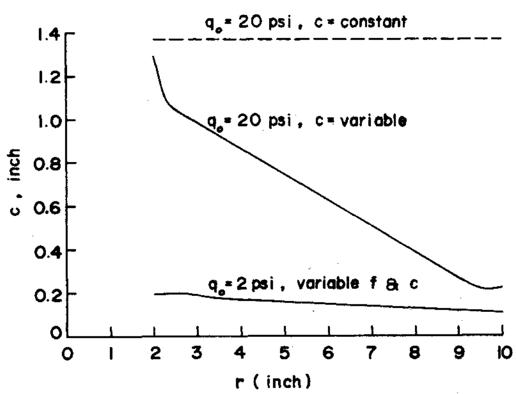

Fig. 15 Core thickness—example 3.

For clarity, the explicit modifications necessary for handling the unknown reaction utilizing the technique described in the previous section are listed following. The state equations become

$$\frac{dx_1}{dr} = x_3 \tag{40a}$$

$$\frac{dx_2}{dr} = -\frac{1}{S}\left[\frac{x_6}{2\pi r} + \frac{q_o(r^2 - r_o^2)}{2r}\right] \tag{40b}$$

$$\frac{dx_3}{dr} = -\left[\frac{x_4}{D} + \frac{\upsilon}{r}x_3\right] \tag{40c}$$

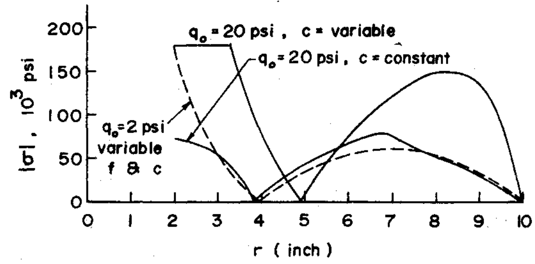

Fig. 16 Bending stress—example 3.

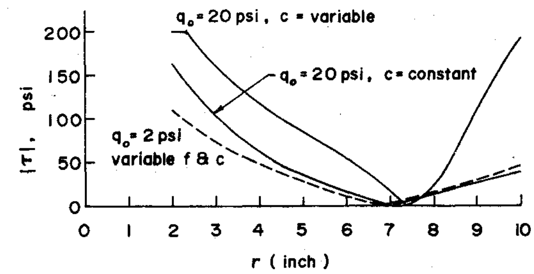

Fig. 17 Shearing stress—example 3.

$$\frac{dx_4}{dr} = -\left[\frac{x_6}{2\pi r} + \frac{q_o(r^2 - r_o^2)}{2r} + \frac{(1-\upsilon)x_4}{r} + D(1-\upsilon^2)\frac{x_3}{r^2}\right] \tag{40d}$$

$$\frac{dx_5}{dr} = 2\pi r(2\rho_1 U_1 + \rho_2 U_2) \tag{40e}$$

$$\frac{dx_6}{dr} = 0 \tag{40f}$$

where as before

$$D = \frac{E_f U_1 (U_1 + U_2)^2}{2(1-\upsilon^2)}$$

$$S = \frac{[G_c (U_1 + U_2)^2]}{U_2}$$

The adjoint (Lagrange multiplier) equations are

$$\frac{d\lambda_1}{dr} = 0 \tag{41a}$$

$$\frac{d\lambda_2}{dr} = 0 \tag{41b}$$

$$\frac{d\lambda_3}{dr} = -\lambda_1 + \frac{\upsilon\lambda_3}{r} + \frac{D(1-\upsilon^2)\lambda_4}{r} \tag{41c}$$

$$\frac{d\lambda_4}{dr} = \frac{\lambda_3}{D} + \frac{(1-\upsilon)\lambda_4}{r} \tag{41d}$$

$$\frac{d\lambda_5}{dr} = 0 \tag{41e}$$

Table 2 Weights and weight savings for example 3

Load q_o (psi)		P (lb)	Weight (lb)	Weight savings (%)
2	c = constant	no solution		no
	c = variable	286.1	0.2761	answer
20	c = constant	2998.6	1.199	50.9%
	c = variable	3250.2	0.5873	

$$\frac{d\lambda_6}{dr} = \frac{\lambda_2}{2\pi Sr} + \frac{\lambda_4}{2\pi r} - \mu_4 \left[\frac{\dfrac{x_6}{2\pi r} + \dfrac{q_o(r^2 - r_o^2)}{2r}}{\pi r(U_1 + U_2)^2} \right] \tag{41f}$$

The controls (U_1 and U_2) are determined from

$$\frac{2\lambda_2}{S(U_1+U_2)}\left[\frac{x_6}{2\pi r}+\frac{q_o(r^2-r_0^2)}{2r}\right]+\frac{\lambda_3 x_4(3U_1+U_2)}{DU_1(U_1+U_2)}$$

$$-\frac{\lambda_4(1-\upsilon^2)x_3}{r^2}\frac{D(3U_1+U_2)}{U_1(U_1+U_2)}+4\pi r\rho_1\lambda_5-\mu_1$$

$$+\mu_3\left[\frac{4x_4^2(2U_1+U_2)}{U_1^3(U_1+U_2)^3}\right]+2\mu_4\frac{\left[\dfrac{x_6}{2\pi r}+\dfrac{q_o(r^2-r_o^3)}{2r}\right]^2}{(U_1+U_2)^3}=0$$

and

$$\frac{\lambda_2(U_1-U_2)}{U_1(U_1+U_2)S}\left[\frac{x_6}{2\pi r}+\frac{q_o(r^2-r_o^2)}{2r}\right]+\frac{\lambda_3 x_4}{D(U_1+U_2)}$$

$$+\frac{2(1-\upsilon^2)D\lambda_4 x_3}{(U_1+U_2)r^2}+2\pi r\rho_2\lambda_5+\mu_2-\frac{2\mu_3 x_4^2}{U_1^2(U_1+U_2)^3}$$

$$-\frac{2\mu_4}{(U_1+U_2)^3}\left[\frac{x_6}{2\pi r}+\frac{q_o(r^2-r_0^2)}{2r}\right]^2=0 \tag{42}$$

It is seen that the boundary conditions are

$$\begin{aligned} &X_1(r)=0 && X_1(r_o)+X_2(r_o)=0\\ &X_2(R)=0 && X_4(r_o)=0\\ &X_4(R)=0 && X_5(r_o)=0 \end{aligned} \tag{43}$$

The transversality conditions are then

$$\begin{aligned} &\lambda_3(R)=0 && \lambda_2(r_o)=-\lambda_1(r_o)\\ &\lambda_5(R)=-1 && \lambda_3(r_o)=0\\ &\lambda_6(R)=0 && \lambda_6(r_o)=0 \end{aligned} \tag{44}$$

The adjoint variables (λ) and the Hamiltonian must be continuous at points of entrance to and exit from the control constraint boundaries. The multipliers μ are chosen such that they are zero when the controls are off of the control boundaries and nonzero when on the control boundaries.

The results of the design for two load intensites ($q_o = 2$, 20 psi) are shown in Figs. (14-17). Note that for load $q_o = 2$, no constant core solution exists. That is, no constant core thickness design, with face thickness equal to or greater than the minimum allowable, results in contact between the inclusion and the support. A summary of the resulting weights and weight savings over constant core designs is shown in Table 2.

V. Conclusions

Sections I-IV have discussed the technique by which the minimum weight design of sandwich plates can be formulated as optimal control problems. The accompanying examples demonstrated that appropriate algorithms can be implemented. It should be noted that in these examples, appreciable weight reduction was achieved through optimum design. This reduction, in some cases, was in excess of 50%.

In general, the optimum designs are rather complex shapes as far as actual construction is concerned. The techniques illustrated in this paper can be used to establish a general guide for suboptimal design as well as a benchmark against which the suboptimal designs can be measured. As more optimum designs are obtained, it may be possible to develop approximate design rules.

The obvious and dramatic weight savings possible in some applications can only be achieved on a large scale when techniques for the economical and reliable fabrication of nonuniform sandwich elements are developed. The results presented in this and similar papers may help spur the development of the necessary technology by pointing out the potential rewards.

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